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Prepared by

H. A. Hassan
Project Coordinator

Department of Mechanical and Aerospace Engineering
Post Office Box 7910
North Carolina State University
Raleigh, North Carolina 27695-7910

During this period work continued on developing turbulence and transition models. A summary of the work is given in two abstracts submitted to 27th AIAA Fluid Dynamics Conference. Copies of these abstracts are enclosed.

During this period two AIAA Papers were presented^{1,2}. The first will appear in the AIAA Journal.

References

1. Robinson, D. F., Harris, J. E. and Hassan, H. A., "A Unified Turbulence Closure Model for Axisymmetric and Planar Free Shear Flows," AIAA Paper 95-0360, January 1995.
2. Baurle, R. A., Alexopoulos, G. A., and Hassan, H. A., "Analysis of Supersonic Combustors with Swept Ram Injectors," AIAA Paper 95-2413, July 1995.

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A $k - \zeta$ (Enstrophy) Model for Compressible Turbulence

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- | | |
|--|--|
| <p>1. Principal Author
Greg Alexopoulos (North Carolina State University)
Research Assistant, Student Member AIAA
Box 7910, NCSU, Raleigh, NC 27695-7910
Tel: 919-515-5671 Fax: 919-515-7968</p> | <p>2.
Hassan A. Hassan (North Carolina State University)
Professor, Associate Fellow AIAA
Box 7910, NCSU, Raleigh, NC 27695-7910
Tel: 919-515-5241 Fax: 919-515-7968</p> |
| <p>3.</p> | <p>4.</p> |

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Concise statement of problem (its genesis and objective):

See attached abstract

Scope and methods of approach, with statement of contribution to the state of the art or an application of existing analytical techniques and theories to a problem:

See attached abstract

Summary of important conclusions:

See attached abstract

Statement of data used to substantiate conclusions, and freehand sketches of major figures to be used (no more than two typed pages):

See attached abstract

A $k - \zeta$ (Enstrophy) Model for Compressible Turbulence

G. A. Alexopoulos *, and H. A. Hassan †
North Carolina State University, Raleigh, NC 27695-7910

October 16, 1995

Abstract

A compressible turbulence model based on the $k - \zeta$ (enstrophy) model of Robinson et al. is developed. The model incorporates a new compressibility correction model that is quite different from previous models. Results are compared to the mixing layer data compiled by Settles and Dodson. It is shown that the predictions of the theory are in good agreement with experiment. Moreover, the present model is superior to the $k - \epsilon$ model with traditional compressibility corrections.

Introduction

Compressible flows require for their description a velocity field and two thermodynamic variables such as the density and temperature. Because of this, the fluctuations of these thermodynamic variables are as important as those of the velocity in determining the resulting turbulent flow. As a result, traditional two-equation and stress models¹ have proven to be inadequate in describing such flows. Therefore, it appears that an appropriate compressible turbulent flow model of the "two-equation" variety, should include six equations that describe variances of velocity, density and temperature and their respective dissipation rates.

A simplification of the above approach can be achieved by invoking Morkovin's hypothesis². According to this hypothesis, the pressure and total temperature fluctuations are small for non-hypersonic compressible boundary layers with conventional rates of heat transfer, i.e.

$$\frac{p'}{P} \ll 1 \quad , \quad \frac{T'_o}{T_o} \ll 1 \quad (1)$$

*Research Assistant, Mechanical and Aerospace Engineering, Student Member AIAA.

†Professor, Mechanical and Aerospace Engineering, Associate Fellow AIAA.

As a result,

$$\frac{\rho'}{\bar{\rho}} \simeq -\frac{T'}{T} \simeq (\gamma - 1)M^2 \frac{u'}{U} \quad (2)$$

In the above equation, ρ' , T' , p' and T' , u' are the fluctuating density, total temperature, pressure, temperature and velocity, M is the Mach number and γ is the ratio of specific heats. The remaining variables represent the mean properties. Based on Eq. (2), equations governing variances of turbulent quantities can be taken as the equation for the turbulent kinetic energy. However, equations governing the dissipation rates of the resulting variances may not be the same.

The present work is based on an extension of the $k - \zeta$ model of Robinson et al.³ to compressible turbulent flows. It is shown in Ref. [3] that the $k - \zeta$ model reproduces all available measurements of growth rates and turbulent shear stress distributions for a variety of free shear layers, and as such, it should provide a good basis for the current model. The compressible enstrophy equation can be written as

$$\begin{aligned} \frac{\partial \zeta}{\partial t} = & 2\overline{\omega'_i \omega'_m} S_{im} + 2\overline{\omega'_i s'_{im}} \Omega_m + 2\overline{\omega'_i \omega'_m s'_{im}} \\ & - 2 \left[\overline{\omega_i'^2} S_{mm} + \overline{\omega'_i s'_{mm}} \Omega_i + \overline{\omega_i'^2 s'_{mm}} \right. \\ & \left. + \overline{\omega'_i u'_m} \frac{\partial \Omega_i}{\partial x_m} + U_m \frac{\partial}{\partial x_m} (\overline{\omega_i'^2} / 2) + \overline{u'_m} \frac{\partial}{\partial x_m} (\overline{\omega_i'^2} / 2) \right] \\ & + 2\epsilon_{ijk} \left[\frac{1}{\bar{\rho}^2} \frac{\partial \bar{\rho}}{\partial x_j} \left(\overline{\omega'_i \frac{\partial p'}{\partial x_k}} - \overline{\omega'_i \frac{\partial \tau'_{km}}{\partial x_m}} \right) + \frac{1}{\bar{\rho}} \overline{\omega'_i \frac{\partial^2 \tau'_{km}}{\partial x_j \partial x_m}} \right] \\ & + 2\epsilon_{ijk} \frac{1}{\bar{\rho}^2} \left(\frac{\partial P}{\partial x_k} - \frac{\partial \bar{\tau}_{km}}{\partial x_m} \right) \overline{\omega'_i \frac{\partial \rho'}{\partial x_j}} \end{aligned} \quad (3)$$

where

$$s'_{ij} = \frac{1}{2} \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right), \quad S_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \quad (4)$$

$$\Omega_i = \epsilon_{ijk} \frac{\partial U_k}{\partial x_j}, \quad \zeta = \overline{\omega'_i \omega'_i}, \quad k = \frac{\overline{u'_i u'_i}}{2}, \quad \tau'_{ij} = -\overline{\rho u'_i u'_j} \quad (5)$$

Here Ω_i and ω'_i are the mean and fluctuating vorticity, $\bar{\tau}_{ij}$ and τ'_{ij} are the laminar and turbulent (Reynolds) stress and ϵ_{ijk} is the permutation tensor.

The Favre-averaged turbulent kinetic energy equation can be written as⁴

$$\begin{aligned} \frac{\partial}{\partial t} (\bar{\rho} k) + \frac{\partial}{\partial x_j} (\bar{\rho} \tilde{u}_j k) = & \tau_{ij} \frac{\partial \tilde{u}_i}{\partial x_j} - \bar{\rho} \epsilon + \frac{\partial}{\partial x_j} \left[\overline{t_{ji} u_i''} - \overline{\rho u_j'' \frac{1}{2} u_i'' u_i''} - \overline{p' u_j''} \right] \\ & - \overline{u_i''} \frac{\partial P}{\partial x_i} + \overline{p' \frac{\partial u_i''}{\partial x_i}} \end{aligned} \quad (6)$$

where t_{ij} and τ_{ij} are the laminar and turbulent (Reynolds) stress. The dissipation rate is

defined as

$$\bar{\rho}\epsilon = \bar{\nu} \left[\overline{\rho\omega_i''\omega_i''} + 2\overline{\rho u_{i,j}''u_{j,i}''} - \frac{2}{3}\overline{\rho u_{i,i}''u_{i,i}''} \right] \quad (7)$$

$$= \bar{\nu} \left[\overline{\rho\omega_i''^2} + \frac{4}{3}\overline{\rho(u_{i,i}'')^2} \right] + 2\bar{\nu} \frac{\partial}{\partial x_i} \left[\frac{\partial}{\partial x_j} (\overline{\rho u_i''u_j''}) - 2\overline{\rho u_i'' \frac{\partial u_j''}{\partial x_j}} \right] \quad (8)$$

The second term in Eq. (8) can be written as

$$2\bar{\nu} \frac{\partial}{\partial x_i} \left[\frac{\partial}{\partial x_j} (\overline{\rho u_i''u_j''}) - 2\overline{\rho u_i'' \frac{\partial u_j''}{\partial x_j}} \right] \simeq 2 \frac{\partial}{\partial x_i} \left[\bar{\nu} \left\{ \frac{\partial}{\partial x_j} (\overline{\rho u_i''u_j''}) - 2\overline{\rho u_i'' \frac{\partial u_j''}{\partial x_j}} \right\} \right] \quad (9)$$

and is normally neglected, consistent with the assumptions of homogeneous turbulence. In this work, the term is included in the diffusion term.

The approach employed in modeling the various terms follows closely that employed in Ref. [3] and will be detailed in the final paper. Moreover, model constants developed in Ref. [3] were unchanged in this model.

The compressibility term in the k -equation has generated a great deal of discussion lately and led to the developments of models by Sarkar et al.⁵, Zeman⁶, Wilcox⁴ and Ristorcelli⁷ among others. None of these models were adopted here because they do not blend smoothly with the incompressible limit. Moreover, there is no reason to assume that the time scale governing this term is dependent on ζ . Instead the term

$$\bar{\nu} \frac{4}{3} \overline{\rho(u_{i,i}'')^2} \quad (10)$$

is modeled as

$$C \frac{k}{\tau_\rho} \quad (11)$$

where C is a model constant and τ_ρ is a time scale. Because the term under consideration is zero when the density is constant, τ_ρ is modeled as

$$\frac{1}{\tau_\rho} = \frac{1}{\bar{\rho}} \sqrt{(\nabla \bar{\rho} \cdot \nabla \bar{\rho}) k} \quad (12)$$

Results and Discussion

To validate the present model, comparisons were made with the two sets of mixing layer experiments that survived the scrutiny of Settles and Dodson⁸. The first set is due to Goebel and Dutton⁹ while the second set is that of Samimy and Elliott¹⁰ and Elliott and Samimy¹¹. Rather than restrict our comparisons to growth rates, we opted to compare the predictions of the theory with the measured velocity, turbulent kinetic energy and turbulent shear stress. Moreover, comparisons were made with a $k - \epsilon$ model using the compressibility correction model proposed by Wilcox. It may be recalled that this model incorporates the best feature

of both Sarkar et al. and Zeman's models. Table 1 summarizes the cases given in Ref. [8] and described in Refs. [9] - [11].

Comparison of present theory with the $k-\epsilon$ model and experiment are shown in Figures 1 - 7. The computational procedure is similar to that employed in Ref. [3]. In presenting the data, we followed the suggestion of Ref. [10] and scaled y with δ_w , the vorticity thickness, defined as

$$\delta_w = \frac{\Delta U}{\left(\frac{\partial u}{\partial y}\right)_{\max}}, \quad \Delta U = U_1 - U_2 \quad (13)$$

where subscripts 1 and 2 represent the fast and slow streams, respectively. Each figure consists of three plots for velocity, turbulent kinetic energy and turbulent shear stress, τ_{xy} . It is seen from the figures that the present theory is in good agreement with cases 2, 3 and 4 of Ref [9] and all cases of Refs. [10] and [11]. For these cases, the $k-\epsilon$ model shows poor agreement with experiment. The agreement is not good with case 5 of Ref. [9]. The reasons for this are not clear at this time. Note that this case is in good agreement with the $k-\epsilon$ model. Furthermore, applying the compressibility correction of Wilcox to the $k-\zeta$ model yields worse agreement with experiment than the $k-\epsilon$ with compressibility corrections.

Settles and Dodson tabulate and plot (see Table II and Fig. 1 of Ref. [8]) normalized mixing layer growth rates versus convective Mach numbers. The data is collected from various sources. As may be seen from the data and figure, there is a great deal of scatter rendering such tabulations and plots meaningless. The reason for the scatter can be traced to a lack of uniformity in calculating and defining growth rates. For most of the available data, the growth rate $db/dx \neq b/x$, where b is the width of the mixing layer. Moreover, there is a lack of uniformity in defining the width of the mixing layer. Because of this, judging the worth of a compressible turbulence theory based on its prediction of growth rates is somewhat misleading.

The final version of the paper will contain the details of model employed and additional comparisons involving wall-bounded flows.

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- [7] Ristorcelli, J. R. "A Pseudo-Sound Constitutive Relationship for the Dilatational Covariances in Compressible Turbulence: An Analytical Theory". ICASE Report 95-22, March 1995.
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- [9] Goebel, S. G., and Dutton, J. C. "Experimental Study of Compressible Turbulent Mixing Layers". *AIAA Journal*, Vol. 29(No. 4):pp. 538–546, April 1991.
- [10] Samimy, M., and Elliott, G. S. "Effects of Compressibility on Characteristics of Free Shear Layers". *AIAA Journal*, Vol. 28(No. 3):pp. 439–445, March 1990.
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Acknowledgments

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Tables

Table 1: Summary of Modeled Experiments

Case, Ref.	U_2/U_1	ρ_2/ρ_1	T_{o1}, T_{o2}	M_1, M_2	M_c
2, [9]	0.57	1.55	578, 295	1.91, 1.36	0.46
3, [9]	0.18	0.57	285, 285	1.96, 0.27	0.69
4, [9]	0.16	0.60	360, 290	2.35, 0.30	0.86
5, [9]	0.16	1.14	675, 300	2.27, 0.38	0.99
1, [10]	0.36	0.64	276, 276	1.80, 0.51	0.51
2, [10]	0.25	0.58	276, 276	1.96, 0.37	0.64
3, [11]	0.25	0.37	276, 276	3.03, 0.45	0.86

Figures

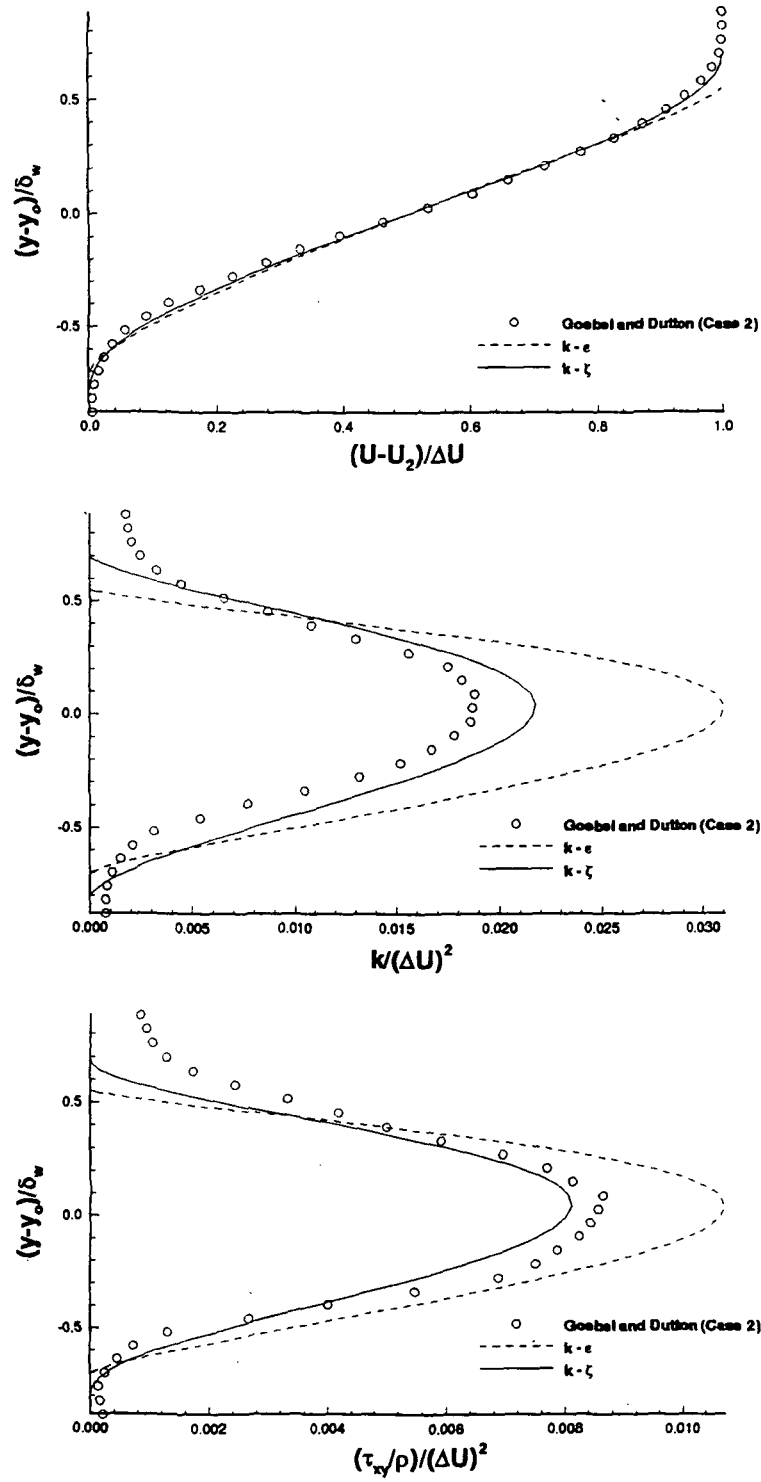


Figure 1: Turbulence Model Comparisons with Experiment (Goebel and Dutton Case 2)

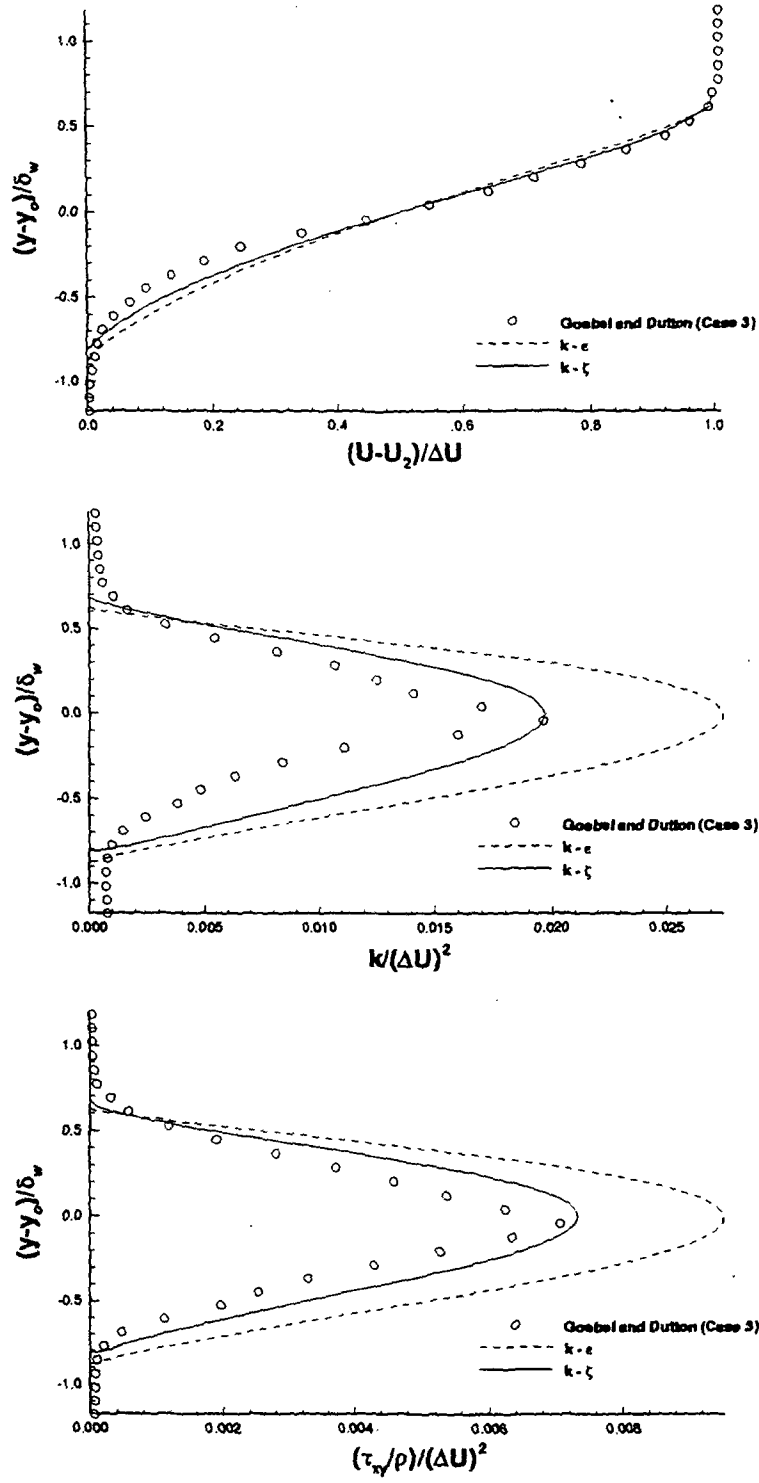


Figure 2: Turbulence Model Comparisons with Experiment (Goebel and Dutton Case 3)

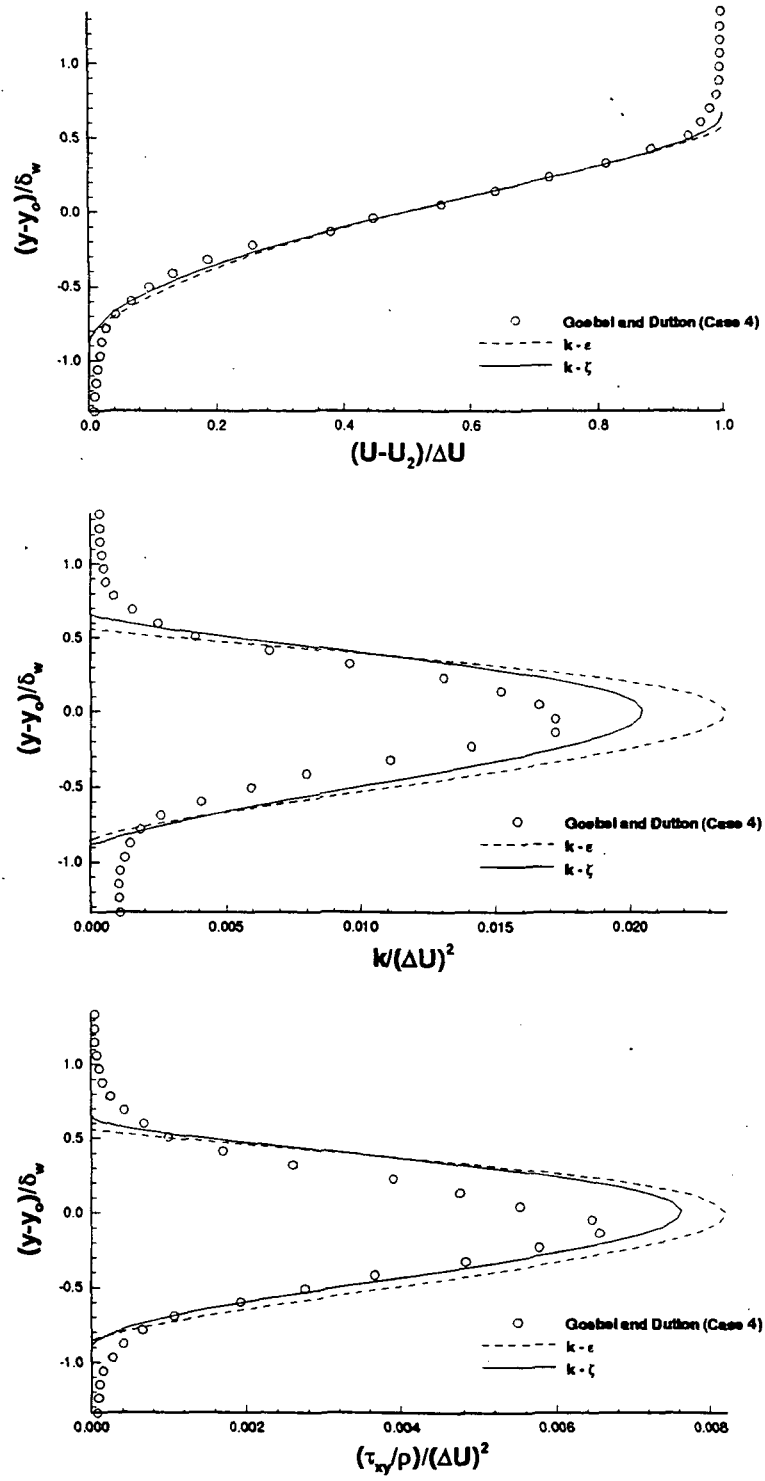


Figure 3: Turbulence Model Comparisons with Experiment (Goebel and Dutton Case 4)

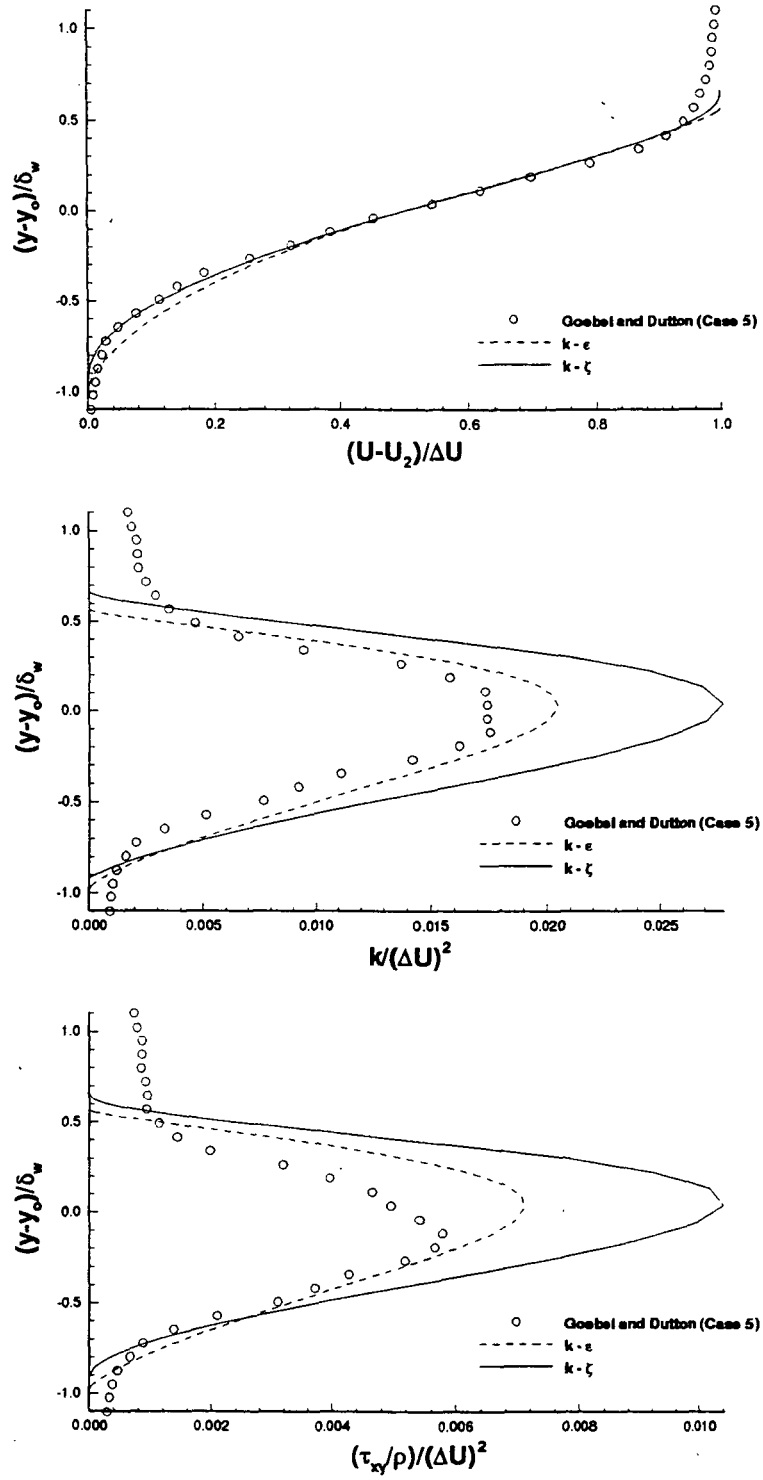


Figure 4: Turbulence Model Comparisons with Experiment (Goebel and Dutton Case 5)

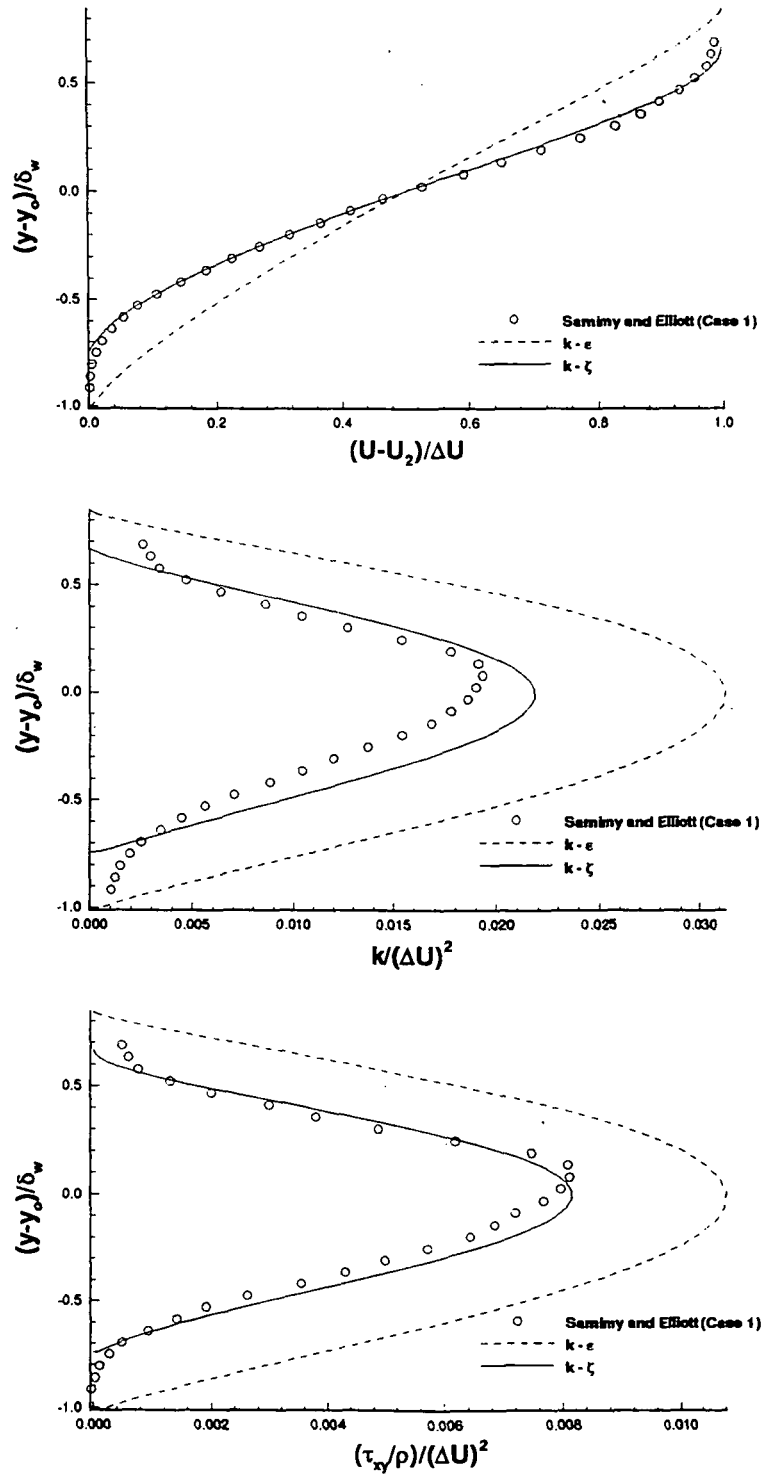


Figure 5: Turbulence Model Comparisons with Experiment (Samimy and Elliott Case 1)

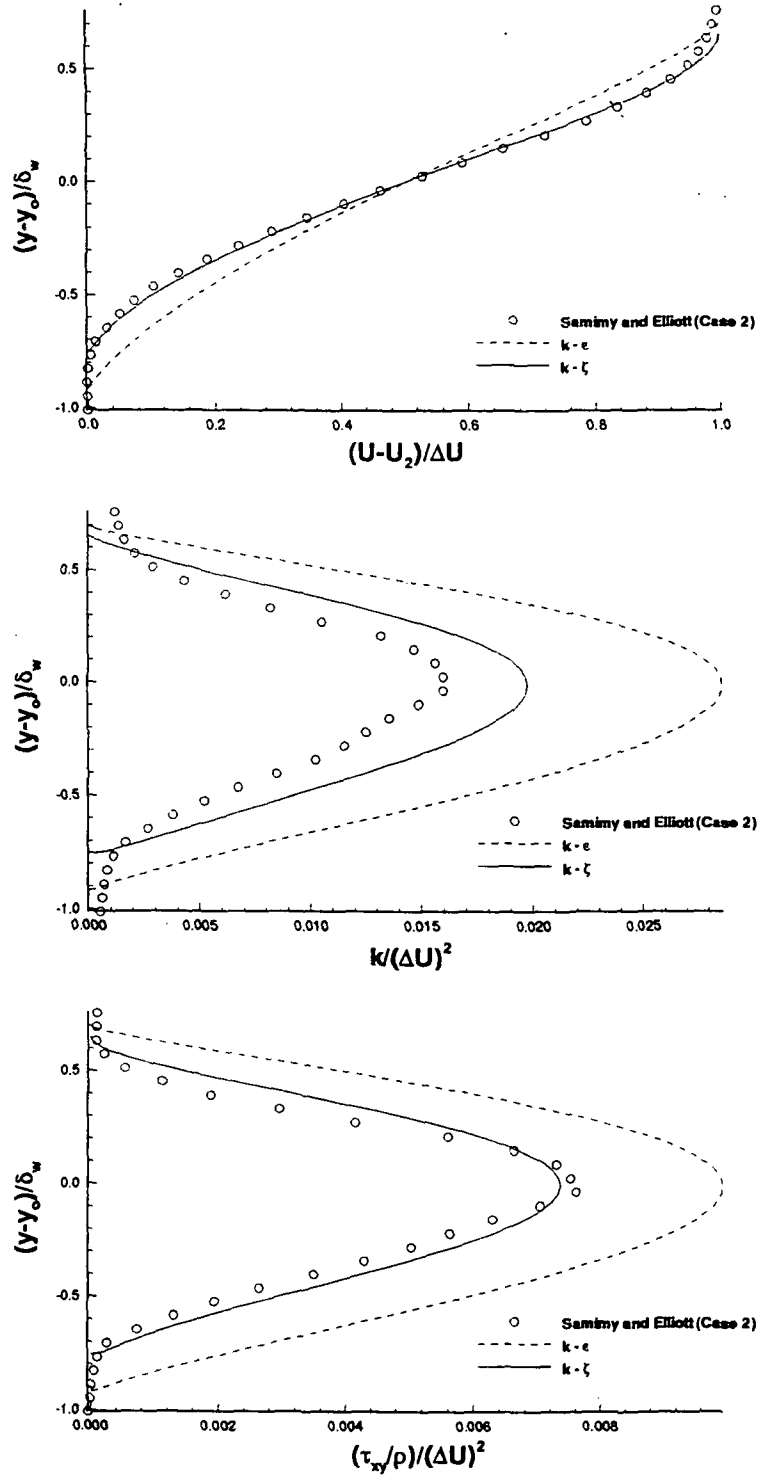


Figure 6: Turbulence Model Comparisons with Experiment (Samimy and Elliott Case 2)

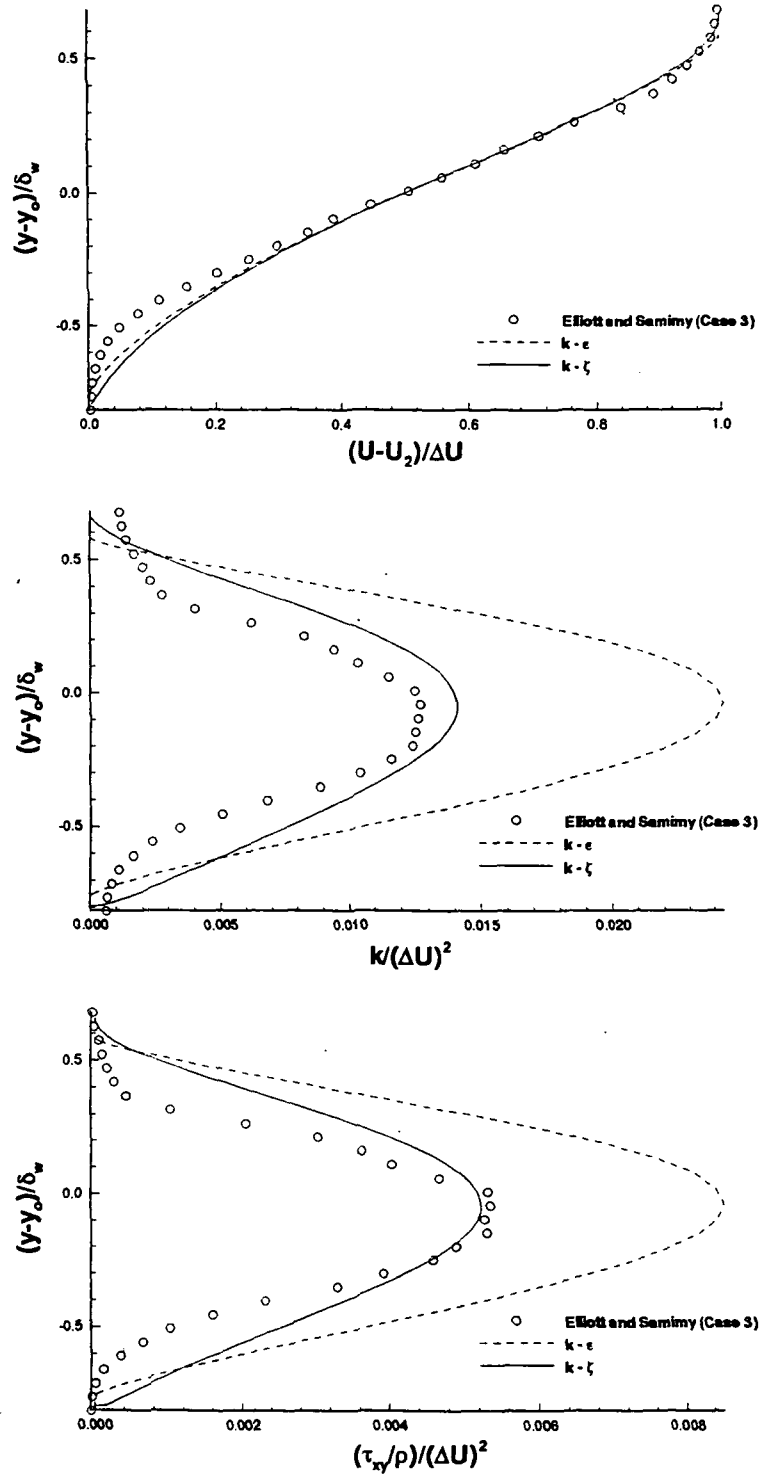


Figure 7: Turbulence Model Comparisons with Experiment (Elliott and Samimy Case 3)

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Paper title (This title will be published in the program; limited to 12 words.):

A Two-Equation Turbulence Closure Model for Wall Bounded and Free Shear Flows

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| <p>1. Principal Author
David F. Robinson (North Carolina State University)
Research Assistant, Student Member AIAA
Box 7910, NCSU, Raleigh, NC 27695-7910
Tel: 919-515-5671 Fax: 919-515-7968</p> | <p>2.
Hassan A. Hassan (North Carolina State University)
Professor, Associate Fellow AIAA
Box 7910, NCSU, Raleigh, NC 27695-7910
Tel: 919-515-5241 Fax: 919-515-7968</p> |
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Summary of important conclusions:

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Statement of data used to substantiate conclusions, and freehand sketches of major figures to be used (no more than two typed pages):

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A Two-Equation Turbulence Closure Model for Wall Bounded and Free Shear Flows

D. F. Robinson *, and H. A. Hassan †

North Carolina State University, Raleigh, NC 27695-7910

October 16, 1995

Introduction

Existing $k-\omega$ and $k-\epsilon$ turbulence closure models are incapable of describing wall bounded and free shear flows using the same set of model constants and boundary conditions. It is generally agreed that the problem with existing models comes from the dissipation equation, ϵ or ω . The reason for this can be traced to the fact that these equations were developed for high turbulent Reynolds numbers, Re_t , but are being employed in situations where Re_t is low.

For most turbulent shear flows Re_t is typically large enough so that one can assume the small scales are nearly independent of the large scales and thus their dissipation rate is isotropic. If one makes this isotropic assumption then the terms in the exact dissipation equation which depend on the mean flow can be neglected. However, for most shear flows, the turbulent Reynolds number varies from very large values in the bulk of the flow to very small values near walls and outside of the boundary layer.

For regions of low Re_t , the small scales of turbulence are weakly dependent on the large scales, and therefore, on the mean flow. Because of this, terms appearing in the exact dissipation equation which depend on the mean flow cannot be neglected.

The above considerations were the basis of a new two-equation model ¹. Instead of modeling the exact equation for dissipation, attention was focused on the enstrophy or the variance of vorticity equation ². The model developed in Ref. [1] was used to describe free shear layers (wakes, jets, and mixing layers). The same set of model constants was used in all calculations. Excellent predictions of growth rates, shear stress, and velocity distributions were obtained.

The object of this work is to use the same set of model constants developed in Ref. [1] to study wall bounded flows. The implementation of the model will proceed in a number of stages. In the first, the model is implemented in a boundary layer code ³ and the results

*Research Assistant, Mechanical and Aerospace Engineering, Student Member AIAA.

†Professor, Mechanical and Aerospace Engineering, Associate Fellow AIAA.

are illustrated by calculating a flow past a flat plate and its wake. Second, the model is implemented in a Navier-Stokes code ⁴ and the results are illustrated by calculating flows past airfoils. The results presented in the abstract are intended to demonstrate that one set of model constants can be used to describe all types of shear flows. More results will be given in the final paper especially those involving adverse pressure gradients.

Analysis

The turbulent kinetic energy and the exact enstrophy or variance of the vorticity equations can be written as

$$\begin{aligned} \frac{D\rho k}{Dt} &= \tau_{im} \frac{\partial U_i}{\partial x_m} - \overline{\mu \frac{\partial u'_i}{\partial x_m} \frac{\partial u'_i}{\partial x_m}} - \frac{\partial}{\partial x_m} \left[\frac{\overline{\rho u'_i u'_i u'_m}}{2} + \overline{p' u'_m} - \mu \frac{\partial k}{\partial x_m} \right] \\ &= \tau_{im} \frac{\partial U_i}{\partial x_m} - \mu \zeta - \frac{\partial}{\partial x_m} \left[\frac{\overline{\rho u'_i u'_i u'_m}}{2} + \overline{p' u'_m} - \mu \epsilon_{ikm} \frac{\partial}{\partial x_k} (\overline{u'_i \omega'_m}) \right] \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{D\zeta}{Dt} &= -2\overline{u'_j \omega'_i} \frac{\partial \Omega_i}{\partial x_j} - \frac{\partial (\overline{u'_j \omega'_i \omega'_i})}{\partial x_j} + 2\overline{\omega'_i \omega'_j s'_{ij}} + 2\overline{\omega'_i \omega'_j S_{ij}} + 2\overline{\omega'_i s'_{ij} \Omega_j} \\ &\quad + \nu \frac{\partial^2 \zeta}{\partial x_j \partial x_j} - 2\nu \frac{\partial \omega'_i}{\partial x_j} \frac{\partial \omega'_i}{\partial x_j} \end{aligned} \quad (2)$$

where,

$$\begin{aligned} s'_{ij} &= \frac{1}{2} \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right) , \quad S_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \\ \Omega_i &= \epsilon_{ijk} \frac{\partial U_k}{\partial x_j} , \quad \zeta = \overline{\omega'_i \omega'_i} , \quad k = \frac{\overline{u'_i u'_i}}{2} \end{aligned} \quad (3)$$

ρ is the density, μ is the viscosity, u'_i and ω'_i are the fluctuating velocity and vorticity and U_i is the mean velocity. Equations (1) and (2) were modeled in Ref. [1] and the results can be written as

$$\begin{aligned} \frac{D\rho\zeta}{Dt} &= -2\frac{\partial \Omega_i}{\partial x_j} \left(\frac{\mu_t}{2\sigma_r} \left[\frac{\partial \Omega_i}{\partial x_j} + \frac{\partial \Omega_j}{\partial x_i} \right] + \frac{\epsilon_{mij}}{2} \left[\frac{\partial (\overline{\rho u'_m u'_i})}{\partial x_l} - \frac{\partial \rho k}{\partial x_m} \right] \right) \\ &\quad + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\zeta} \right) \frac{\partial \zeta}{\partial x_j} \right] - \frac{\rho \beta_5}{R_k} \zeta^{\frac{3}{2}} + \left(\alpha_3 \zeta b_{ij} + \frac{2}{3} \delta_{ij} \zeta \right) \rho S_{ij} \\ &\quad - \frac{\zeta \tau_{ij} \Omega_i \Omega_j \beta_4}{k \Omega} + \epsilon_{ilm} \left(\frac{\tau_{ij}}{k} \right) \left(\frac{\partial k}{\partial x_l} \frac{\partial \zeta}{\partial x_m} - \frac{\partial k}{\partial x_m} \frac{\partial \zeta}{\partial x_l} \right) \frac{\Omega_j}{\Omega^2} \beta_8 \\ &\quad - \frac{2\beta_6 \tau_{ij} \nu_t}{k \nu} \Omega \Omega_i \Omega_j + \frac{\beta_7 \rho \zeta}{\Omega^2} \Omega_i \Omega_j S_{ij} \end{aligned} \quad (4)$$

$$\frac{D\rho k}{Dt} = \tau_{im} \frac{\partial U_i}{\partial x_m} - \mu\zeta + \frac{\partial}{\partial x_m} \left[\left(\frac{\mu}{3} + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_m} + \frac{\mu_t b_{mj}}{\rho \sigma_p} \frac{\partial P}{\partial x_j} \right] \quad (5)$$

where

$$\tau_{ij} = -\overline{\rho u'_i u'_j} = \mu_t \left(2S_{ij} - \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) - \frac{2}{3} \rho k \delta_{ij} \quad (6)$$

$$\nu_t = \frac{\mu_t}{\rho}, \quad R_k = \frac{k}{\nu \sqrt{\zeta}} \quad (7)$$

with μ_t being the eddy viscosity. The model constants are tabulated in Table 1.

Two departures from earlier modeling should be noted. The free shear flows considered in Ref. [1] were all characterized by a constant pressure. In the proposed applications the pressure gradient is not zero. The pressure fluctuations appear explicitly in the k but not the ζ equations. Therefore, the turbulent diffusion term in the k -equation is modeled as

$$- \left[\frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x_m} + \frac{\mu_t b_{mj}}{\rho \sigma_p} \frac{\partial P}{\partial x_j} \right] \quad (8)$$

where

$$b_{mj} = \frac{\tau_{mj}}{\rho k} + \frac{2}{3} \delta_{mj} \quad (9)$$

and σ_p is a model constant.

The second adjustment concerns the dissipation term in the ζ equation. The turbulent time scale $\frac{k}{\nu\zeta}$ can not be less than Kolmogorov's time scale. To allow for this result, the dissipation term now has the form

$$\frac{\rho \beta_5 \zeta^{\frac{3}{2}}}{R_k + \delta} \quad (10)$$

For the results presented here $\delta = 0.1$.

Results

For high turbulent Reynolds numbers, the eddy viscosity is chosen as

$$\nu_t = \frac{C_\mu k^2}{\nu \zeta}, \quad C_\mu = 0.09 \quad (11)$$

For bounded flows the eddy viscosity and the turbulent Reynolds number are both zero at the wall. Because of this, the above expression is traditionally multiplied by the function f_μ . There is no unanimity on the form of f_μ in the near wall region⁵. More recent implementations such as that of Speziale et al⁶ employ a function that depends on both Re_t and y^+ where

$$Re_{et} = \frac{k^2}{\nu^2 \zeta}, \quad y^+ = \frac{y U_\tau}{\nu}, \quad U_\tau = \sqrt{\frac{\tau_w}{\rho}} \quad (12)$$

where τ_w is the wall shearing stress. Because separation may take place in the presence of adverse pressure gradients a different representation that does not require y^+ is employed. The resulting expression can be expressed as

$$f_\mu = \min(f_\mu^*, 1.0) \quad , \quad f_\mu^* = \left(1 + \frac{C_{\mu_1}}{Re_t^{\frac{3}{2}}}\right) \exp\left(\frac{-\sqrt{k}y}{\nu C_{\mu_2}}\right) \quad (13)$$

and

$$C_{\mu_1} = 5.0 \quad , \quad C_{\mu_2} = 40.5 \quad (14)$$

The boundary conditions for the two-equation model are

$$\begin{aligned} k_w &= 0 \\ k_\infty &= \frac{3}{2}(TU_\infty)^2 \\ \frac{\partial \zeta}{\partial y} \Big|_w &= 0 \quad , \quad \text{or} \quad \frac{\partial}{\partial y} \left(\frac{1}{3} \frac{\partial k}{\partial y} \right) = \zeta \quad \text{at} \quad y = 0 \end{aligned} \quad (15)$$

where T is the turbulent intensity ($\approx 1\%$) and ζ_∞ is determined by specifying a free stream ratio of $\frac{\mu_t}{\mu} \Big|_\infty$ along with k_∞ . For the current work we are using a first order difference for $\frac{\partial \zeta}{\partial y} \Big|_w$ which amounts to simply extrapolating ζ to the wall.

All of the results presented here use the first boundary condition for ζ . Comparisons with the other boundary condition will be given in the final paper.

There are a number of tests a turbulence model must meet to be considered a successful model. First, it must predict the correct friction and pressure coefficients and other near wall measurements ⁵ of shearing stress, kinetic energy, and dissipation. Second, it must predict the correct growth rate, asymptotic shearing stress distribution, and velocity in the far wake. Finally, it must predict the correct 'B' constant that appears in the expression for the velocity in the log-law region, and the manner in which k varies with y in the near wall region [See Table 4.5 in Ref. [7]].

Figures 1- 3 show plots of k^+ , ζ^+ , and $(-\overline{\rho u'v'})^+$ vs. y^+ in the near wall region where

$$k^+ = \frac{k}{U_\tau^2} \quad , \quad \zeta^+ = \frac{\zeta \nu^2}{U_\tau^4} \quad , \quad -\overline{\rho u'v'} = \frac{\tau_{1,2}}{U_\tau^2} \quad (16)$$

Also shown is the "average" of available data ⁵ together with the measurements of Schubauer ⁸ and Laufer ⁹. As seen from these figures, the predictions compare well with the results summarized in Ref. [5] and experiments. It appears from Figure 1 that the peak k^+ is under predicted, nevertheless, it lies within the range of available experimental data ($k^+ \approx 2.8 - 6.0$). Figure 4 compares the calculated skin friction coefficient with the correlations of Cole and Schoenherr ¹⁰. Again good agreement is indicated.

The constant 'B' in the log-law correlation

$$u^+ = \frac{1}{\kappa} \ln y^+ + B \quad (17)$$

is typically 5. Current calculations show the 'B' is not exactly constant, but rather, varies slightly from approximately 4.7 to 5.2 over a y^+ ranging from 50 to 500.

The next set of figures compares calculated and measured wake data. Figures 5- 7 show a comparison of numerical, asymptotic, and experimental results for the defect velocity at the centerline, $\frac{U_e}{W_o}$ [U_e is the edge velocity, W is the defect velocity], wake half width, b , and the maximum $\left(\frac{\tau}{W_o^2}\right)$, where τ is the shear stress, as a function of $\frac{x}{\theta}$ where θ is the momentum thickness. The asymptotic solution is that of the far wake with a constant eddy viscosity. The experimental results are obtained from Pot ¹¹ and Weygandt and Mehta ¹². Again, good agreement is indicated.

Figures 8- 9 compare the defect velocity, W , and the shear stress distributions across the wake. Here, also good agreement with experiment ^{11, 13} is indicated.

The remaining figures show the Navier-Stokes solution for a NACA 0012 airfoil. Figure 10 shows the grid (239x101) which has an initial normal spacing of $2.e^{-6}\bar{C}$ and an outer boundary of $15\bar{C}$. Figures 11- 12 show the pressure and skin friction coefficients compared with the experimental data ¹⁴ and with the standard $k - \omega$ model of Wilcox ⁷. Once again, good agreement with experiment is shown.

Concluding Remarks

From the results indicated here, it appears that the model developed in Ref. [1] can be used for wall bounded shear flows and it has also been shown that the model can be incorporated into either boundary layer or Navier-Stokes codes. The next task is to test the model for a variety of other flows including separated flows.

Acknowledgment

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Table 1. Enstrophy Equation Model Constants

Constants	$k - \zeta$
C_μ	0.09
κ	0.41
α_3	0.35
β_4	0.42
β_5	2.37
β_6	0.10
β_7	0.75
β_8	2.30
σ_r	2.00
$\frac{1}{\sigma_k}$	1.80
$\frac{1}{\sigma_\zeta}$	1.46
C_{μ_1}	5.0
C_{μ_2}	40.5
δ	0.1

Figures

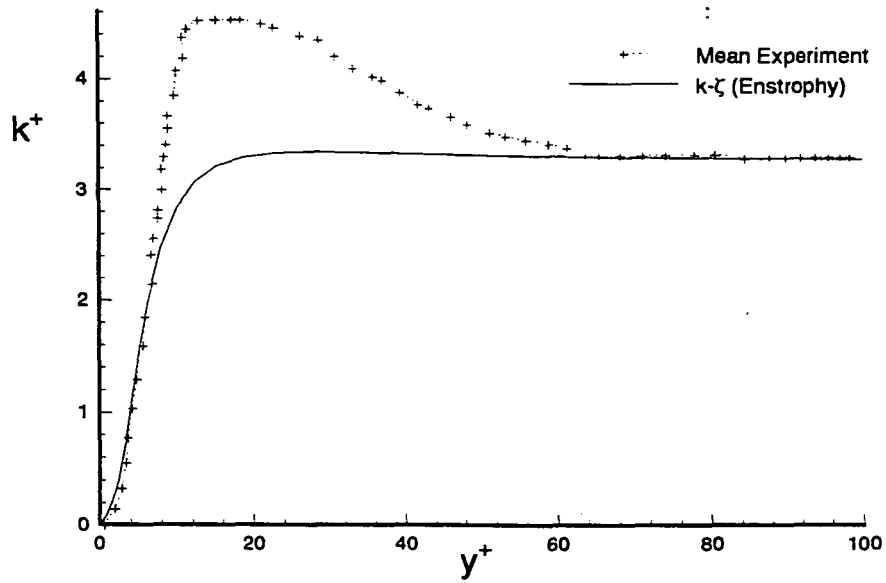


Figure 1. Near Wall Behavior of Turbulent Kinetic Energy (Flat Plate)

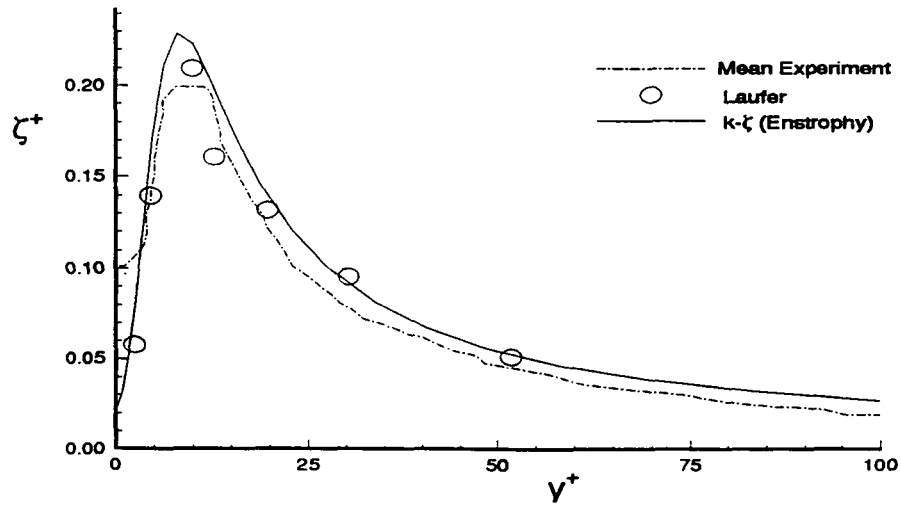


Figure 2. Near Wall Behavior of Enstrophy (Flat Plate)

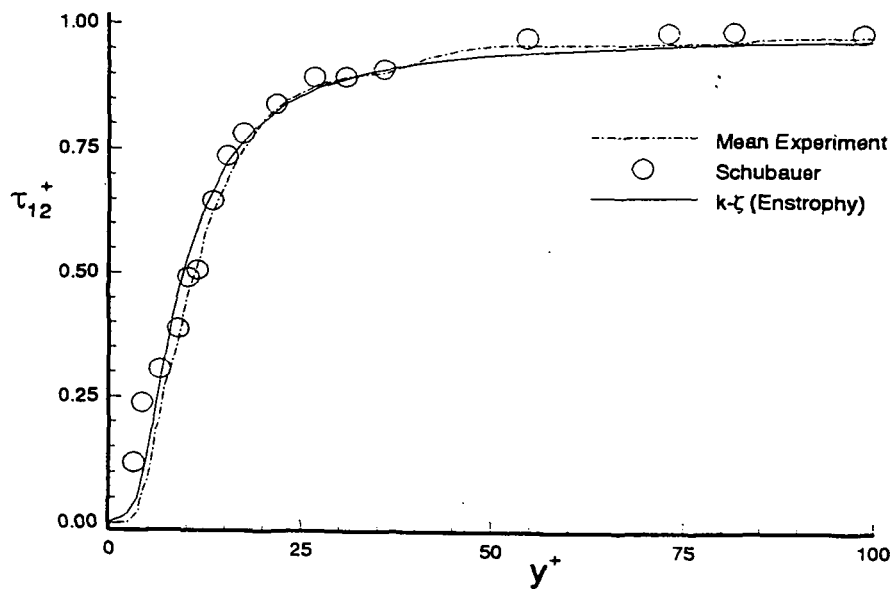


Figure 3. Near Wall Behavior of Reynolds Stress (Flat Plate)

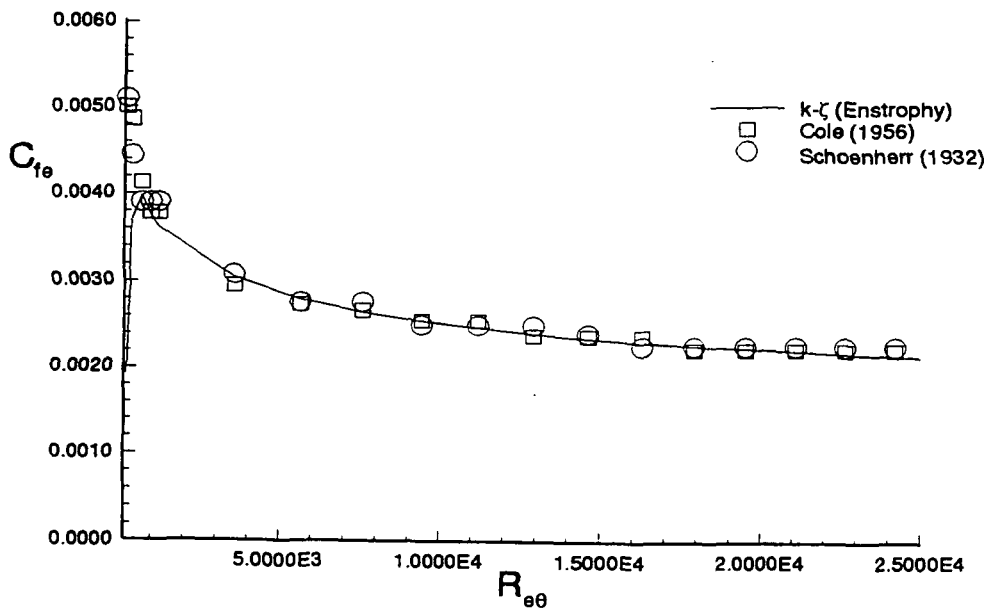


Figure 4. Skin Friction Coefficient (Flat Plate)

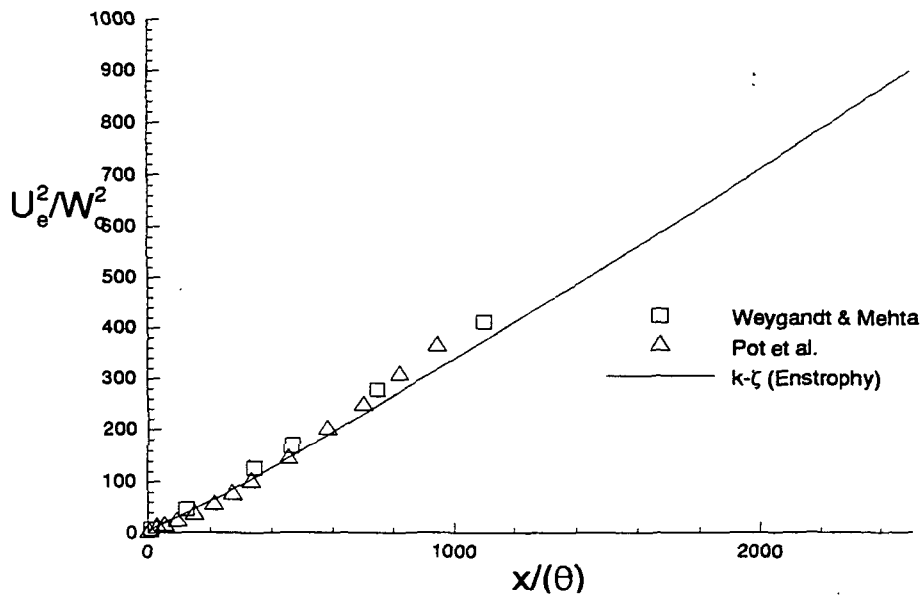


Figure 5. Streamwise Variation of Maximum Velocity Defect

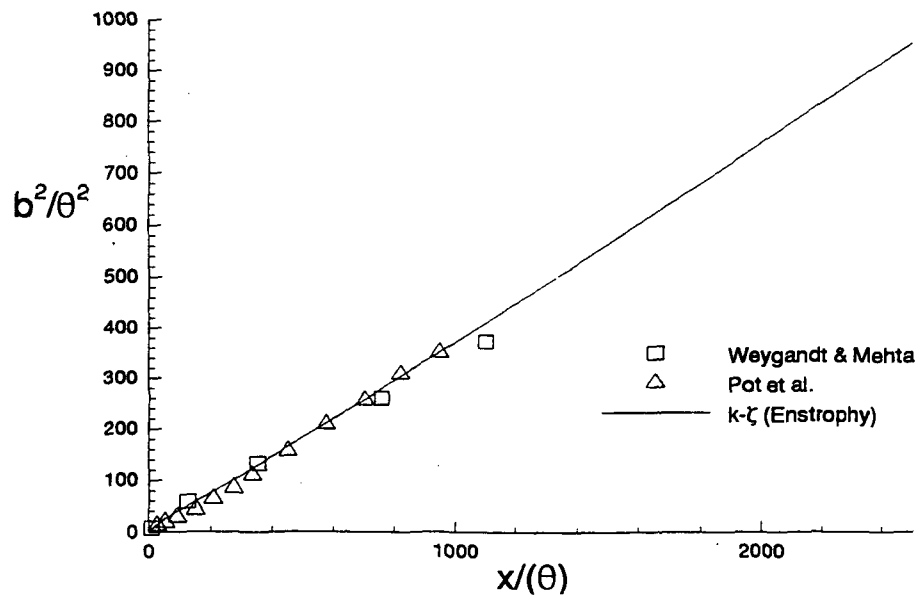


Figure 6. Streamwise Variation of Wake Half Width

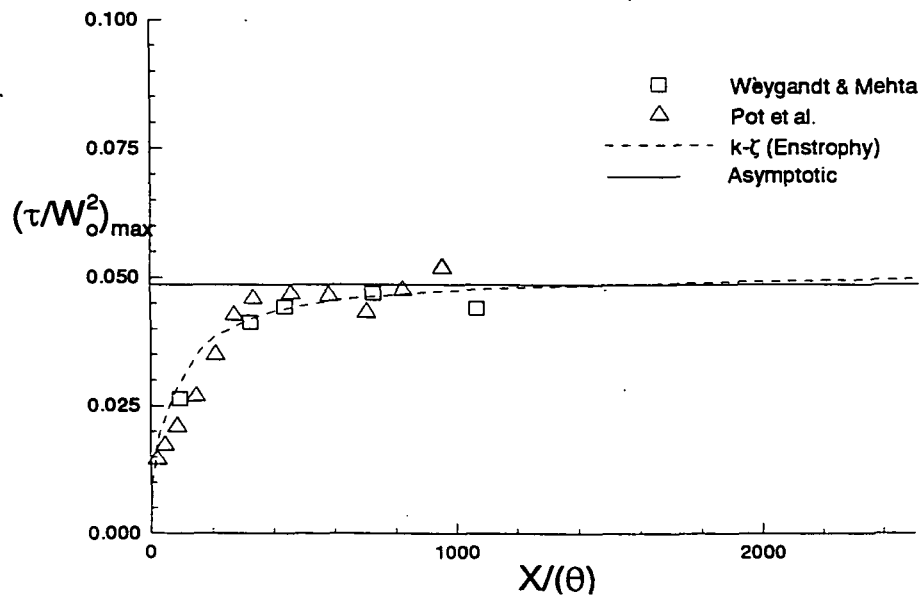


Figure 7. Streamwise Variation of Maximum Shear Stress

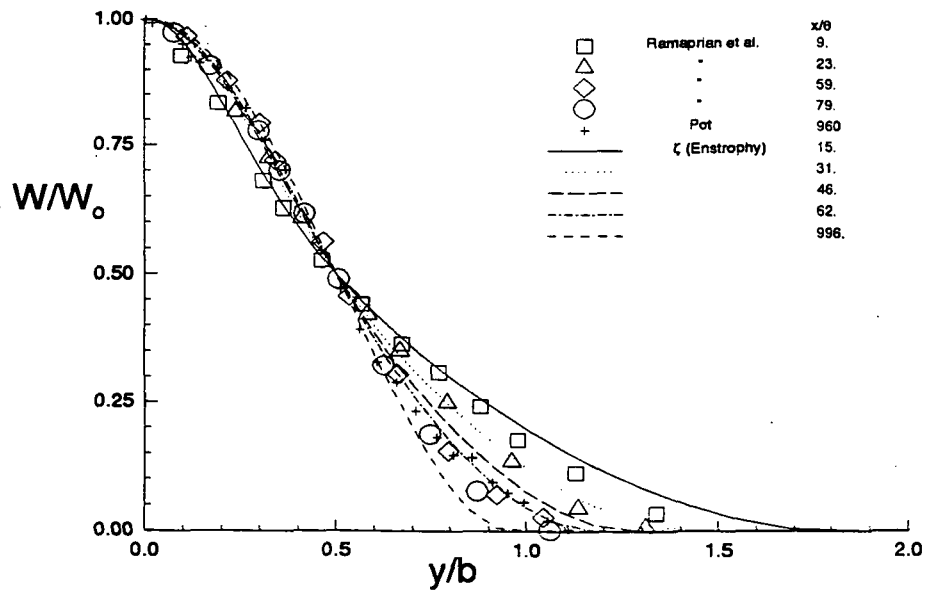


Figure 8. Wake Velocity-Defect Profiles

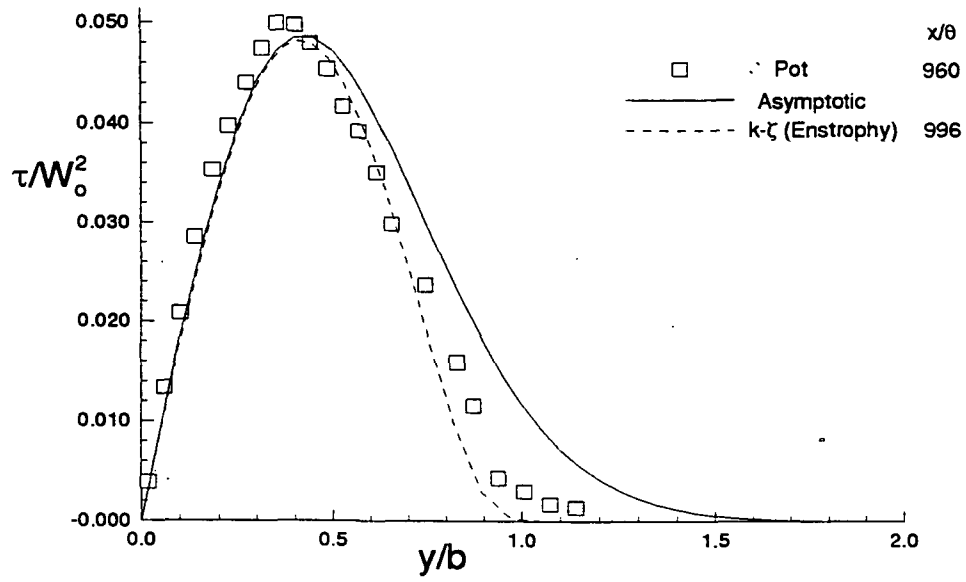


Figure 9. Wake Shear Stress Profiles

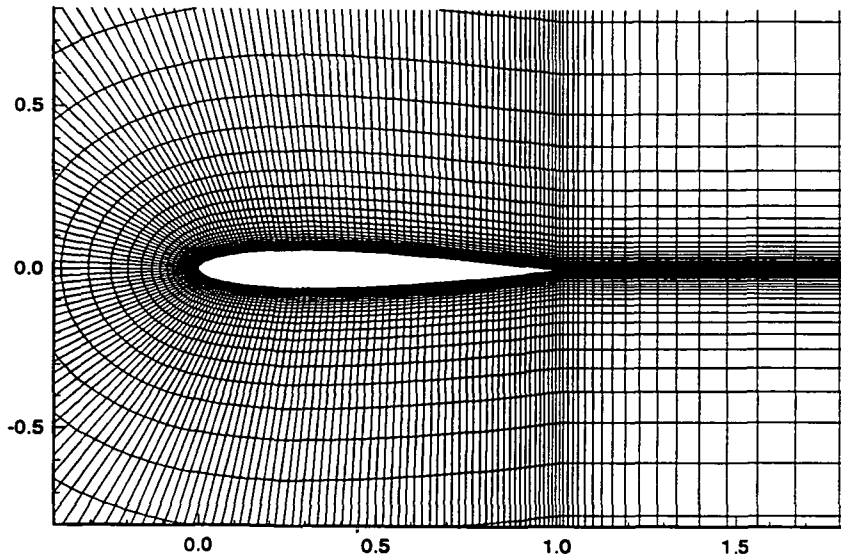


Figure 10. Partial View of C-mesh for NACA 0012 Airfoil

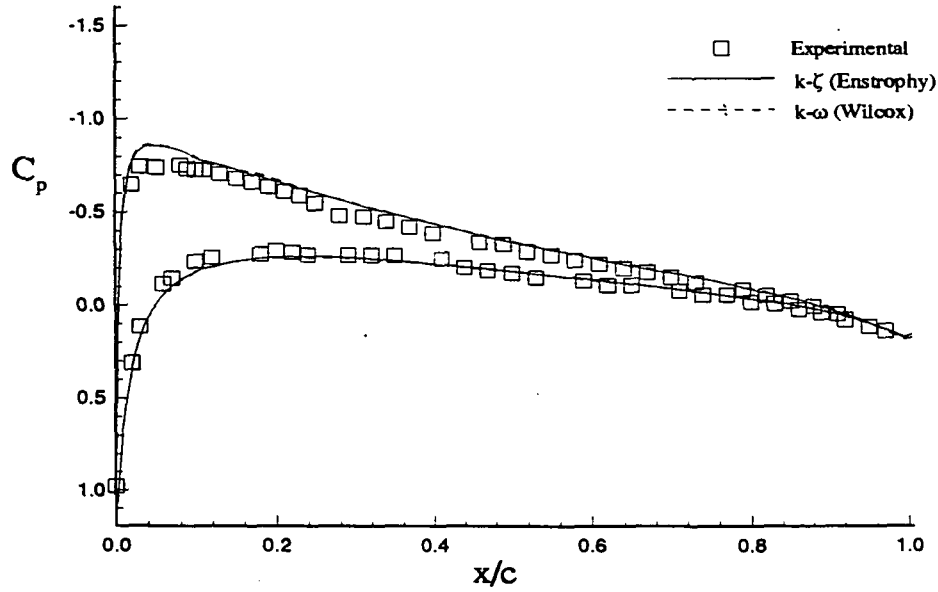


Figure 11. Pressure Distribution for NACA 0012 Airfoil ($M_\infty = .502$, $Re_\infty = 2.91e6$, $\alpha = 2.06$)

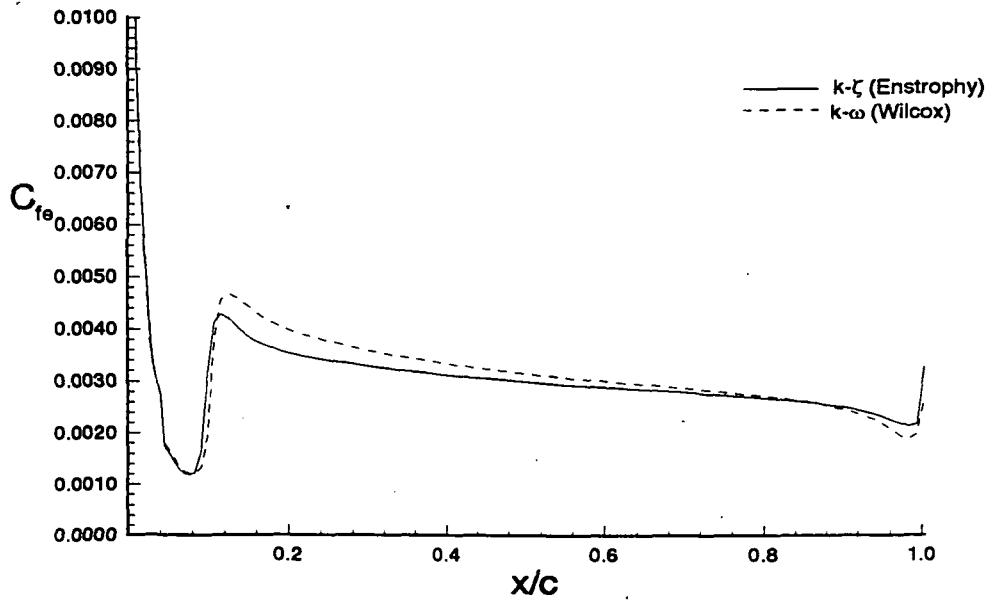


Figure 12. Skin Friction Comparison with $k-\omega$ for NACA 0012 Airfoil ($M_\infty = .502$, $Re_\infty = 2.91e6$, $\alpha = 2.06$)